# The medieval roots of modern scientific thought. A Fibonacci abacus on the facade of the church of San Nicola in Pisa 

Pietro Armienti<br>Dipartimento di Scienze della Terra, Università di Pisa, Via Santa Maria 53, Pisa, Italy

## ARTICLE INFO

## Article history:

Received 17 March 2015
Accepted 24 July 2015
Available online xxx

## Keywords:

Fibonacci
Pisa
Intarsia


#### Abstract

A marble intarsia on the main entrance of the church of San Nicola in Pisa provides the opportunity to appreciate the level of cultural excellence achieved by the Maritime Republic at the height of its power during the twelfth and thirteenth centuries. The intarsia reveals the direct influence of the great Pisan mathematician Leonardo Fibonacci due to the presence of circles whose radii represent the first nine elements of the Fibonacci's sequence and which were arranged to depict some properties of the sequence. Moreover, the tiles can be used as an abacus to draw sequences of regular polygons inscribed in a circle of given radius. This construction is a novelty that has resurfaced after eight hundred years of neglect and its implications, in themselves, are worthy of special examination. The presence of so many symbolic references makes the intarsia an icon of medieval philosophical thought and reveals aspects that pave the way to modern scientific thought.


© 2015 Elsevier Masson SAS. All rights reserved.

## 1. Introduction and aim of the work

Time had made indecipherable, for centuries, a marble intarsia on the facade of the church of San Nicola in Pisa, obscuring the brilliant intuitions that gave birth to a school of thought that would overturn the medieval worldview and make Pisa the cradle of modern scientific thought. The message, inscribed for posterity in the lunette of the main portal of the church, has re-emerged in all its detail thanks to its recent restoration, which brought the marble back to its original splendour (Fig. 1) allowing recognition of the intarsia as one of the main monuments of the town.

## 2. Leonardo Pisano and its age

Son of the merchant Bonaccio, a customs officer of the Republic, Leonardo Pisano also named Fibonacci (=filius Bonacci) was born in Pisa in 1175. He was called by his father to Bugia, the modern day Béjaïa in Algeria, to learn the art of trade. In those days, Bugia was a port with lively contacts around all the Mediterranean, so Leonardo could learn the basics of arithmetic and geometry of the ancient Greeks, whose ideas the Arabs, unlike the Western Christians, were studying and developing. Being the son of a merchant, Leonardo quickly realized that working out profits or goods in stock was much simpler with the decimal numbering system of the Arabs,

[^0]http://dx.doi.org/10.1016/j.culher.2015.07.015
1296-2074/® 2015 Elsevier Masson SAS. All rights reserved.
who in turn had learned it in their trade with the Indians. As a result, together with goods and profits, Leonardo brought the decimal numbering and a passion for arithmetic and geometry back to Pisa and the West.

In those years, the best thinkers of the West were engaged in subtle theological disputes, while an ancient notion of Pope Gregory the Great who, in the climate of the declining Roman Empire, still prevailed. He had ruled that faith was to be praised over intelligence, and that belief without the benefit of the proof was to the credit of the faithful. Those were the times when Anselm of Aosta, founder of scholastic philosophy, was forced to justify himself for his demonstration of the existence of God, by saying that even idleness of the mind could generate vices and that, therefore, one could, in fact one ought to, reason so as not to err.

Leonardo had grown up in this climate, and among the theological discussions that oiled the mechanisms of the balancing act between the two superpowers of the time, the Church and the Empire, he had introduced intellectual speculation on more practical aspects of reality, those which provided more lucrative opportunities than theology. His fame was linked to some relevant works that he produced at the beginning of the thirteenth century:

- De Practica Geometriae, (Practice of Geometry). A book on geometry based on Euclid's "Elements" and "On Divisions";
- Liber Abaci, (The Book of Calculating). An encyclopaedia of thirteenth century mathematics, both theoretical and practical.


Fig. 1. (a) The state of the intarsia in the 1980s and its position on the ancient main portal left of the present entrance. (b) 2015 photograph showing the restored lunette. The arrow points to the detail discussed in this essay.

One of the problems in Liber Abaci involves the famous sequence $1,1,2,3,5,8,13 \ldots$ with which his name is irrevocably linked (Quot paria coniculorum in uno anno ex uno paro germinentur: "How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?"). Actually, it was much later ( $\sim 1870$ ) that Edouard Lucas named this famous series of numbers after Fibonacci [1].

Fibonacci's reputation as a mathematician was so great that Frederick II summoned him for an audience when he was in Pisa around 1225. Virtually nothing is known of Fibonacci's life after that date, except that by decree the Republic of Pisa awarded the 'serious and learned Master Leonardo' a yearly salary of 'libre XX denariorem' in addition to the usual allowances, to reward Fibonacci for his pro bono advising to the Republic on accounting and related mathematical matters.

## 3. The Fibonacci sequence

In his treatise Liber Abaci of 1210 [2], Fibonacci had proposed a numerical sequence in which the first term, the number 1 , is united with itself to generate the second, and all other terms are derived from the sum of the previous two:
-1-2-3-5-8-13-21-34-55...

This sequence appears in many natural patterns, such as the unfolding of the spiral shels of gastropods, the arrangement of leaves on a stem, or the distribution of clouds of interstellar dust: phenomena in which the state of a system depends on the sum of previous conditions [3].

It exhibits several properties, among which:

- if we divide any number $\left(\mathrm{F}_{\mathrm{N}}\right)$ by the second preceding it in the sequence ( $\mathrm{F}_{\mathrm{N}-2}$ ) we always get two as a result $\left(\mathrm{F}_{\mathrm{N}} / \mathrm{F}_{\mathrm{N}-2}=2\right)$ plus an advance equal the number just before the divisor ( $\mathrm{F}_{\mathrm{N}-3}$ )
$\frac{F_{N}}{F_{N-2}}=2+F_{N-3}$
(e.g.: 55/21 $=2$ remnant = 13);
- the sum of the first $\mathrm{N}-2$ consecutive elements of the series plus 2 is the $N$ number of the series
$\sum_{1}^{N-2} F_{i}+2=F_{N}$
(e.g.: for $\mathrm{N}=6: 1+2+3+5+2=13$ );
- the ratio between two successive terms of the series tends to the golden ratio 1.618...

It is widely accepted that Fibonacci treated this series only lightly, practically only in relation to the problem of the rabbits. However, I will show that this view is probably not true, and that he and his disciples were perfectly aware that by exploring a sequence which, starting from one, unfolds in infinite and varied multiplicity, just as creation was called into existence by the creative will of the original One, they could also open up new avenues not only to geometry, but to theology itself.

## 4. The Golden Ratio

The golden ratio, which artists and architects in classical antiquity regarded as the canon of beauty in a work of art, is renamed 'the divine proportion' in the Christian West. The golden ratio is an irrational number that cannot be calculated as the ratio of two integers, but rather represents the ratio of two quantities which are such that one is the mean proportion between the other and their sum: in the case of a rectangle with sides $\mathbf{a}$ and $\mathbf{b}$, these are said to be in golden proportion if the following relation holds:
$\mathbf{a}: \mathbf{b}=\mathbf{b}:(\mathbf{a}+\mathbf{b})$
This rule reveals the intimate harmony of the realities produced by a simple repetition of events, which continually generate others, like the union of two lovers. From two contiguous values of the series, $\mathbf{a}$ and $\mathbf{b}$, the sum $\mathbf{c}$ follows, carrying with it the effects of $\mathbf{a}$ and $\mathbf{b}$, its originators: $\mathbf{c}=\mathbf{a + b}$. But in the constant evolution of creation $\mathbf{b}$ and $\mathbf{c}$ can also join and generate $\mathbf{d}=\mathbf{b}+\mathbf{c}$, in an infinite sequence that contains the fulfilment of the relationship between $\mathbf{a}$ and $\mathbf{b}$, from which they are derived and from whose merging they come into existence.

## 5. The intarsia on the main entrance Church of San Nicola

The intarsia shows the elegant symmetries of a delicate arabesque, which seems testament to the frequent contacts between the Maritime Republic of Pisa and the Arab world [4]. But the message written in the marble goes well beyond this. The tangle of features characterized by regular symmetries produces polygonal shapes that are clearly visible in the soaring octagonal bell tower next to the facade. In addition, the pilasters marking out the tower sides present a square section at the base of the building and a pentagonal one beyond the string-course of the first level; the top drum rises to the sky like a minaret, and the hexagonal belfry rests on sixteen arches that form twice as many sides as the base. From the inside, the belfry can be reached via a spiral staircase supported


Fig. 2. Representation of the first 9 elements of the Fibonacci sequence based on the increasing diameters of the circles found in the intarsia.
by arches that progress upwards in a heptagonal fashion. However, the intarsia presents symmetries that reproduce regular polygons with 4,8 and 16 sides, and does not appear to feature any polygons with 5,6 or 7 sides.

Focusing on the background, one notices that the arabesque is inscribed within a circle, which is inscribed within a square, which is in turn inserted in a rectangle whose ratio of the longer side to the side of the square is the golden ratio (Fig. 1B). The golden ratio recurs in the grid of the background, now fully visible after the restoration, and it is precisely the background that provides the key to understanding the meaning of the lunette and its prominent position on the facade.

The dating of the building appears controversial; according to some sources it dates back to 1281 [5] and could be attributed to Nicola Pisano. However, this date is forty years after the death of Fibonacci. Furthermore, Nicola Pisano was born around 1225 and arrived in Pisa from Apulia as a fully trained sculptor [6]. Therefore, attributing the bell tower and the church of San Nicola to the architect Nicola Pisano would lead to the hypothesis that the intarsia may be the work of a group of students of Fibonacci. The attribution to the architect Diotisalvi, a contemporary of Fibonacci, would date the building astride the twelfth and thirteen centuries [5], thus placing the intarsia under the direct influence of the great Pisan mathematician.

The intarsia makes explicit reference to the Fibonacci series and its properties, some of which are known to have been discovered centuries after construction of the intarsia, but were evidently well known to the author of its design who adopts the first nine elements of the Fibonacci series to organize the abacus. Fig. 2 shows a geometric modelling of the intarsia. A scaled photo of the intarsia is shown as a faded fill of the diagram area (arbitrary units).

If we assume that the smallest circles have radii $=1$, radii of the next larger circles are twice as large, and those of the subsequent circles are three times as large.

Those with a radius 5 are subdivided into sectors and placed in the small squares at the corners of the square, which encloses the main circle. The central circle has a radius of 13 , while the circles inscribed in the small squares in the corners, have a radius of 8 . The other elements in the intarsia are arranged in tracks that form circles of radius 21 and 34, and finally the circle that circumscribes the intarsia has a radius 55 times larger than the smallest circle. 1,
$2,3,5,8,13,21,34,55$ are the first nine elements of the Fibonacci sequence: the reference could not be more explicit, directly connecting the intarsia to the work of the great mathematician or a group of his collaborators or students. Moreover, the concentric coronas forming a girdle in the intarsia have maximum and minimum radii of 21 and 13 respectively (Fig. 2) and have their centres on the circle of diameter 34. This arrangement describes the property that $\mathrm{F}_{\mathrm{N}} / \mathrm{F}_{\mathrm{N}-2}=2$ with the advance of $\mathrm{F}_{\mathrm{N}-3}$. In fact, all the circles of size 21 in the girdle (the external blue outer limit of the girdle) are tangent both to the circle 55 and to the circle of size 13 in the centre. With reference to the diameters this implies: $110=42+42+26$ that is equivalent to: $55=21+21+13$ (or $55 / 21=2$ advance $=13$ ). The same rule applies to circles of radii 34 and 13 , in fact $34=13+13+8$ (there are four coronas of size 2 , between the two circles of radius 13 , whose centres lie at the extremes of the double arrowed green line that is also the radius of circle 34 ).

The size of the line borders in this way is necessarily 2 , as required by the fact that the difference between 21 and 13 is 8 , and has to be distributed on four coronas of equal size (see in Fig. 2 the concentric arrangement of circles of radii between 13 and 21 of the girdle).

The other linear elements of the intarsia (Fig. 2: red profiles of battlements and of the two central crossed squares) are inscribed in circles of $55-2=53$ respectively ( 2 is the width of all the coronas), and $34-2=32$.

Fifty-three and 32 are the sums of the $\mathrm{N}-2$ Fibonacci's numbers that precede 55 and 34 in the series. This arrangement is related to the second property stated above for the Fibonacci's series.

Lower limits of the battlements are arranged on the circle of radius $34-2$, while their cusps are defined by a kink marked by the circle of radius $42=55-13$ along which the circles of radii 1 and 2 are also aligned.

Some different considerations, linked to the grid in the lunette around the intarsia, demonstrate the awareness of the author of the design of the relations between the Fibonacci's series and the golden ratio. This allowed him to find and represent to a very good approximation, regular polygons inscribed in a circle of a given radius r .

With reference to Fig. 3, one can begin with the observation that in the rectangle XYIZ , the ratio of side XY to side XZ is the golden ratio $\phi$. In fact $\mathrm{XY}: \mathrm{XZ}=\mathrm{XZ}:(\mathrm{XZ}+\mathrm{XY})$ since in the arbitrary units of the figure $\mathrm{XY}=110, \mathrm{YI}=178$ and, consequently, $\mathrm{AO}=55$. Thus: $\mathrm{XZ}=\phi^{*} \mathrm{XY}$ where $\phi=1.618034 \ldots$. XY is also the side of the central square, in which a circle of radius $r=X Y / 2=A O$ is inscribed.

From the image, it is possible to infer that the right-angled triangles on YI have a hypotenuse equal to $\mathrm{r} \phi$. Therefore:
$\mathrm{EI}=\mathrm{r} \phi-\mathrm{r}=\mathrm{r}(\phi-1)=\mathrm{r} \tau$
where $\tau=\phi-1$ is the golden part of the unit and EI the golden part of the radius. Note that among other properties, $\phi$ is such that $1 / \phi=\phi-1$ :

Euclid's Elements (Book II. Prop. 11-8,9) [7] contained the demonstration that the golden part of $r$ corresponds to the side of the decagon inscribed in a circumference whose radius is $r$ (let us call it $\mathrm{S}_{10}$ ). Therefore, the intarsia in the facade of the church of San Nicola contains all the data to build both the decagon and the pentagon, since the side $S_{5}$ of the pentagon is the chord subtended by an arc twice that of the chord subtended by the side of the decagon.

But Fig. 1b contains more information. The round dot indicated by the arrow in Fig. 1B, is placed in the middle of the catethus of one of the triangles on side YI (Fig. 3, point M) and, when taken in relation to the strange closure shaped like a squashed "M" one can see at the limit of the lunette (at the back of the arrow, Fig. 1), it


Fig. 3. The circular frame of the intarsia is scaled to fit a square of size 110 in arbitrary units. This allows to preserve the correct ratios among all the elements of the intarsia whatever be the final size of the figure. In the formula $\mathrm{D}^{\prime} \mathrm{D}=\sqrt{2} \mathrm{CD}=$ $\sqrt{2\left(R^{2}-\left(\frac{R\left(\tau-\varphi^{n}-1\right.}{\sqrt{2}}\right)^{2}\right.}-\left(R\left(\tau-\varphi^{n}-1\right)\right) \mathrm{R}$ is the radius of the green circle, $\Phi$ is the golden ratio $(1,618034 \ldots), \tau^{-} \phi-1$ and $\boldsymbol{n}$ is the number of times that EI is to be segmented to find the golden part of the golden part (see text).
suggests a projection at an angle of $45^{\circ}$. A similar shape in a symmetrical position suggests the same projection from the opposite side. When we carry out the projections on the side XZ of the points $E$ and $I$ and their symmetrical points, with respect to $G, E^{\prime}$ and $Y$ (Fig. 3), it becomes immediately apparent that the position and the sides of the two interlocked squares at the centre of the intarsia are correlated to $\mathrm{S}_{10}$, and their diagonals equal $2^{*} \mathrm{~S}_{10}$. Their side is, therefore, equal to $2 \frac{\pi \mathrm{r}}{\sqrt{2}}$ while the radius of the inscribed circle is equal to half this value. Surprisingly, the circumference of the circle inscribed in the external white outline of the interlocked squares at the centre corresponds to the value:
$\frac{2 \pi \tau}{\sqrt{2}} \quad \mathrm{r}=\sqrt{2} \pi \tau \quad \mathrm{r}=\frac{\sqrt{2}}{\varphi} \pi \quad \mathrm{r}$
That formula contains the product of the main irrational numbers known in antiquity linked to the problems of the golden ratio, the squaring of the circle $(\pi)$ and the incommensurability of the side and the diagonal of the square $(\sqrt{2})$.

Let us consider now the intersections of the projections of F and its symmetrical F', with the circumference (D' and D, Fig. 3). We can now interpret the continuation of the pattern of the background toward the lower part of the lunette as an indication to continue the partitioning of EI according to the golden principle and to project the segments obtained onto the circumference of radius $r$. For the first partition (Fig. 3)
$\mathrm{EF}=\mathrm{GI}-\mathrm{FI}-\mathrm{GE}=\mathrm{R} \phi-\mathrm{R} \tau^{2}-\mathrm{R}=\mathrm{R}\left(\phi-\tau^{2}-1\right) ;$
for the $\mathrm{n}^{\text {th }}$ repetition $\mathrm{R}\left(\phi-\tau^{\mathrm{n}}-1\right)$
where $R$ is a positive integer which, at most, assumes the value of $r(R \leq r)$. From EF we get:
$\mathrm{BC}=\mathrm{EF} / \sqrt{2}=\mathrm{R} \phi-\tau^{n}-1 / \sqrt{2}$,
and from the rectangle triangle BOC considering that $\mathrm{BO}=\mathrm{BC}$ :
$\mathrm{AB}^{2}=\mathrm{AO}^{2}-\mathrm{BO}^{2}=\mathrm{R}^{2}-\left(\left(\mathrm{R}\left(\phi-\tau^{\mathrm{n}}-1\right)\right) / \sqrt{ } 2\right)^{2}$

Table 1

| R | $n$ | Side | M1 this method | M2 Chord theorem | \|M2-M1| |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 55 | 2 | 5 | 63.71 | 64.70 | 0.99 |
| 55 | 3 | 6 | 53.88 | 55.00 | 1.12 |
| 55 | 4 | 7 | 47.35 | 47.70 | 0.35 |
| 55 | 9 | 9 | 37.04 | 37.60 | 0.56 |
| 54 | 5 | 8 | 41.60 | 42.10 | 0.50 |
| 53 | 9 | 10 | 33.90 | 34.00 | 0.10 |
| 51 | 9 | 11 | 30.73 | 31.00 | 0.27 |
| 50 | 11 | 12 | 28.44 | 28.50 | 0.06 |
| 48 | 9 | 13 | 25.90 | 26.30 | 0.40 |
| 47 | 9 | 14 | 24.27 | 24.50 | 0.23 |
| 46 | 9 | 15 | 22.64 | 22.90 | 0.26 |
| 45 | 12 | 17 | 20.09 | 20.20 | 0.11 |
| 44 | 10 | 18 | 18.86 | 19.10 | 0.24 |
| 43 | 9 | 19 | 17.64 | 18.10 | 0.46 |
| 43 | 10 | 20 | 17.18 | 17.20 | 0.02 |
| 42 | 9 | 21 | 15.94 | 16.40 | 0.46 |
| 42 | 10 | 22 | 15.47 | 15.70 | 0.23 |
| 42 | 13 | 23 | 14.90 | 15.00 | 0.10 |
| 41 | 9 | 24 | 14.22 | 14.40 | 0.18 |
| 41 | 10 | 25 | 13.75 | 13.80 | 0.05 |
| 41 | 13 | 26 | 13.16 | 13.30 | 0.14 |
| 40 | 9 | 27 | 12.48 | 12.80 | 0.32 |
| 40 | 10 | 28 | 12.00 | 12.30 | 0.30 |

All values in the table are in arbitrary units. The table reports the sides of the regular polygons that can be inscribed in a circle of radius $r=55$. Sides are obtained with the recursive geometric procedure illustrated in Fig. 3, translated in the formula $\sqrt{2\left(R^{2}-\left(\frac{R\left(\tau-\varphi^{n}-1\right.}{\sqrt{2}}\right)^{2}\right.}-\left(R\left(\tau-\varphi^{n}-1\right)\right)$. For each side value, the table provides the values of $\mathrm{R}(\leq \mathrm{r})$, $n$ and the exact value provided by the chord theorem that relates chord lengths with arc lengths: chord $=2^{*} \mathrm{r}^{*} \sin (\alpha / 2)$. In regular polygons $\alpha=2 \pi / \mathrm{m}$ were $m$ is a positive integer.

Since $\mathrm{BD}=\mathrm{AB}$ we obtain:

$$
\begin{aligned}
\mathrm{CD}= & \mathrm{BD}-\mathrm{BC}=\mathrm{AB}-\mathrm{BO}=\sqrt{\left(R^{2}-\left(\frac{R\left(\varphi-\tau^{n}-1\right)}{\sqrt{2}}\right)^{2}\right.} \\
& -\left(\frac{R\left(\varphi-\tau^{n}-1\right)}{\sqrt{2}}\right)
\end{aligned}
$$

and from the rectangle triangle $D^{\prime} C D$
$D^{\prime} D=\sqrt{2} C D=\sqrt{2 *\left(R^{2}-\left(\frac{R\left(\varphi-\tau^{n}-1\right)}{\sqrt{2}}\right)^{2}\right.}-\left(R\left(\varphi-\tau^{n}-1\right)\right)$
This last equation represents the algebraic version of the geometric procedure adopted in the abacus, and for $\mathrm{R}=55$ and $n=1$, provides for D'D a value of 77.78, exactly the expected size of the square inscribed in the circle of side 55 . For $\mathrm{R}=55$ and $n=4$, D'D $=47.35 \ldots$ which differs from the side 47.72 of the regular heptagon by less than $1 \%$.

In this way, the construction of regular polygons occurs according to a recursive procedure that involves the construction of circles centred in the centre of a golden rectangle whose upper side is to be divided into golden parts and can provide the values of the sides of regular polygons inscribed in the circle of radius $\mathbf{r}$. The construction of the polygons with 5, 6, 7, 8, 9 and 17 sides is shown in Fig. 4. For the heptadecagon in Fig. 4, note that the circle on which we make projections following the golden ratio procedure is that of radius 45 . The switch to smaller circles produces the lengths of polygons with a larger and larger number of sides.

The constructions in Fig. 4 also show that the sides of the polygons are marked by elements of the abacus that the green vertical dashed lines help to illustrate and that, when the abacus was conceived, were probably directly recognizable to make polygons.


Fig. 4. Graphical representation of the use of the abacus for the construction of regular polygons. The formula $\sqrt{2\left(R^{2}-\left(\frac{R\left(\tau-\varphi^{n}-1\right.}{\sqrt{2}}\right)^{2}\right.}-\left(R\left(\tau-\varphi^{n}-1\right)\right)$ provides the value $S_{m}$ of the side of a polygon with m sides that can be inscribed in a circle of maximum radius r . The formula is the algebraic version of the geometric procedure adopted in the abacus. Note that the formula provides approximate solutions for regular polygons inscribed in circles of radius $r$ only if $R$ assumes integer values $\leq r$. The green vertical dashed lines help to recognize the elements of the abacus that are related to the sides of regular polygons: a: pentagon $R=55, n=2$, $S_{5}=63.71$ (arbitrary units) (from the chord theorem: $64,7=2 \operatorname{Rsin}(360 / 5 / 2)$ ) The table indicates the exact values obtained with a trigonometric calculation, and the corresponding error; b: hexagon; c: heptagon; d: octagon; e: ennagon; f: heptadecagon. For heptadecagon note that the value of the side has been obtained using $\mathrm{R}=45$ and $n=12$. Values of polygons from 5 to 28 sides inscribed in a circle of radius 55 are reported in Table 1

This algorithm, that allows us to calculate the sides of the regular polygons inscribed in the largest circle of the intarsia, does not conflict with the famous demonstration of Gauss of the impossibility of constructing, by means of compass and unmarked straightedge only, regular n-gone for which $n$ is not the product of a power of 2 and any number of distinct Fermat primes. In fact, the abacus procedure provides only approximate solutions. Table 1 summarizes some of the obtained results.

## 6. Discussion

Symptomatic of the cultural climate of the Republic, which at the time controlled the trade in the Mediterranean, the lunette could well be said to represent a milestone in the history of scientific thought of the Christian West. In fact, the arabesque is not a mere ornament or a collection of geometrical shapes; rather it is a signature, the acknowledgment of the Wisdom of the Creator whose perfect Universe conforms to recognisable rules. The position of the intarsia, on the main entrance of the church, is an expression of the thesis supported by Thomas Aquinas in his Summa Theologiae, i.e.
that knowledge is a gateway to the divine, and rational truth and revealed truth cannot contradict one another:
"Veritas: Adaequatio intellectus ad rem. Adaequatio rei ad intellectum. Adaequatio intellectus et rei." [8].

In the Summa, Aquinas assigned to theology the role of making the fundamentals of faith transparent to reason: this position would be openly adopted three centuries later by Galileo, when he stated [9]:
"The human understanding can be taken in two modes, the intensive or the extensive. Extensively, that is, with regard to the multitude of intelligibles, which are infinite, the human understanding is as nothing even if it understands a thousand propositions; for a thousand in relation to infinity is zero. But taking man's understanding intensively, in so far as this term denotes understanding some proposition perfectly, I say that the human intellect does understand some of them perfectly, and thus in these it has as much absolute certainty as Nature itself has. Of such are the mathematical sciences alone; that is, geometry and arithmetic, in which the Divine intellect indeed knows infinitely more
propositions, since it knows all. But with regard to those few which the human intellect does understand, I believe that its knowledge equals the Divine in objective certainty, for here it succeeds in understanding necessity, beyond which there can be no greater sureness."

Galileo Galilei
In this context, the architecture of Romanesque churches becomes the collective tool for the education of the people and the expression of their participating in Revealed Truth; as a result architectural forms should convey perfectly harmonic proportions.

In the years when San Nicola was being built, Leonardo Fibonacci, having long abandoned his merchant guise, was now an intellectual whose students and disciples formed a well-respected circle, capable of offering theology their own interpretation of the world. Fibonacci's methods and solutions could be applied to problems of arithmetic and geometry, yet they could also penetrate the nature of the divine. And this is the reason for the place of honour given to the intarsia at the main entrance of the church on Via Santa Maria. At the end of the thirteenth century, in Pisa, with a lead of three centuries over Galileo, Fibonacci is fully aware of the value of intellectual scientific speculation and offers theologians the opportunity to understand the nature of the divine through thoughts about the world. In the temple of Via Santa Maria there were church people willing to embrace his positions.

## 7. Conclusions and open problems

The presence of so many symbolic references makes the intarsia an icon of medieval philosophical thought and reveals aspects that pave the way to modern scientific thought. This monument was meant for the education of elites, in tune with the aims of scholastic philosophy: a precious gift of the wisdom of the ancients whose heritage must be valued.

In creating this masterpiece, artists, theologians, mathematicians and artisans worked closely together, following a common code based on the insights of Fibonacci, and with utter dedication to their art. I believe I have deciphered only a small part of this code, and many historiographic and mathematical problems remain open. For example the intarsia shows that the artists were fully aware of the connections that existed between their plan, the Fibonacci sequence and the golden ratio, even though, until today, the discovery of these connections has been attributed to Luca Pacioli, a mathematician of the early seventeenth century [1]. When exactly was the intarsia designed? Should it be attributed directly to the influence of the great mathematician or is it a tribute to his
greatness by students and collaborators? What was the context in which the ideas of Fibonacci were shared and what role did they play in the foundation of the University in Pisa? From a mathematical perspective, several aspects still await explanation, for example the role of all the elements of the abacus in the background, or if the rotation or the change of symmetry in some elements of the bell tower are embellishments designed to give an air of movement to the structures or rather code messages.

At the end of this exploration, which began one summer morning in front of the church while grumbling about the delay of my wife, I am left with the belief that it is always worth waiting for the preparations of a beautiful woman. One may be rewarded with a hidden treasure.

## Acknowledgements

I would like to thank my colleagues at Pisa Univesity and Scuola Normale Superiore for their support in discussions on this topic, so removed from my usual research interests: Gianfranco Fioravanti, Marco Collareta, Marina Soriani, Antonio Albano, Anna Santoni. The traslation into English was due to the joint efforts of Monica Boria and Wendy Doherty. And last but not least, thanks to Dave Westerman for his final reading of this paper.

## References

[1] D. Fowler, Some episodes in the life and times of Division in Extreme and Mean Ratio (233-248), in: Atti del Conv. intern di studi: "Luca Pacioli e la matematica del Rinascimento", Sansepolcro, 1994.
[2] E. Bombieri, L'eredità di Fibonacci nella teoria dei numeri (35-43), in: Fibonacci tra arte e scienza, A cura di L. A. Radicati di Brozolo. Ed. cassa di risparmio di Pisa, 2002.
[3] M. Livio, The Golden Ratio The Story of Phi, the World's Most Astonishing Number, Broadway Books, New York, 2002, ISBN 0-7679-0815-5, 304 pp.
[4] A. Milone, "Arabitas" pisana e medioevo mediterraneo. Relazioni artistiche tra XI e XIII secolo (101-132), in: Fibonacci tra arte e scienza, A cura di L. A. Radicati di Brozolo. Ed. cassa di risparmio di Pisa, 2002.
[5] E. Tolaini, Forma Pisarum. Storia urbanistica della città di Pisa. Problemi e ricerche, Seconda edizione riveduta e accresciuta, Ed Nistri-Lischi, Pisa, 1979, pp. 1979.
[6] F. Rodi, Da Biduino a Nicola "de Apulia", Geometria e calcolo nell'architetura al tempo di Leonardo Fibonacci, 2002, pp. 69-84.
[7] R. Migliorato, G. Gentile, Euclid and the scientific thought in the third century B.C, Ratio Math. 15 (2005) 37-64.
[8] F. Amerini, Tommaso d'Aquino, la verità e il Medioevo". Quaestiones de veritate q. 1, a. 1, ad 3, Annali del Dipartimento di Filosofia (Nuova Serie) XV (2009) 35-63.
[9] G. Galileo, Dialogo sopra i massimi sistemi del mondo (trans. by Stillman Drake, Dialogue Concerning the Two Chief World Systems), University of California Press, Berkeley, 1953.


[^0]:    E-mail address: armienti@dst.unipi.it

